Digital Capacitance System for Mass, Volume, and Level Measurements of Liquid Propellants

Robert L. Blanchard* and Arthur E. Sherburne†
Trans-Sonics, Inc., Burlington, Mass.

A method of using a digital capacitance to balance a bridge electronically has been applied to the measurement of quantities of propellants. Solid state switches connect into the bridge that combination of "*n*" capacitors which most nearly balances the bridge. The capacitors have values proportional to the binary scale. The measurement is directly in binary digital code that is easily converted to multiple scale analog output. The response of the systems to a step input is rapid (about one bit per millisecond), linear, and without overshoot. The system is conveniently adaptable to time sharing of the sensor; an important application of which occurs in propellant utilization control. The electronic circuits are adaptable to automatic checkout and are interchangeable with a minimum of adjustment. Applications of the method to propellant quantity measurements include the measurement of mass independent of temperature and temperature gradient and the measurement of level and volume independent of density. The mass measurement is particularly suited to pure dielectric fluids such as hydrogen and oxygen. The measurement of level and volume is suitable for fluids having a wide range of dielectric constants and has been generalized for application to conducting fluids. The system is flexible in application, accurate, free from moving parts, insensitive to acceleration, and suitable for use in rocket environments.

Nomenclature

ω = angular frequency radians
ϵ_0 = dielectric constant free space
ϵ = relative dielectric constant
ρ = density
C = capacitance
M = mass
k = constant
h = liquid height
H = sensor height

Subscripts

L = liquid
g = gas
ρ = density
o = oxidizer
= fuel
m = measuring
r = reference
v = in vacuum

I. Introduction

In large liquid propelled rockets, an important measurement is that of propellant quantity. The functions for which quantity information is required are loading (tanks nearly full) and inflight measurement for flight information and PU control (simultaneous depletion).

Techniques of measurement in which a capacitance or a change in capacitance is proportional to propellant quantity have offered very suitable properties but with certain important limitations. This paper presents a new method of performing the capacitance measurement which offers a solution to many of these limitations and which has important advantages for accurate, reliable measurements in rocket environments.

Presented at the ARS 17th Annual Meeting and Space Flight Exposition, Los Angeles, Calif., November 13–18, 1962; revision received September 12, 1963.

* Vice President and Director, Research and Engineering Division.

† Manager, Electronics Laboratory.

II. Concept

Accurate measurements of electrical quantities such as capacitance are usually performed by a bridge circuit in which the unknown is measured by adjusting a variable known element until the bridge is balanced. In the system that is described here, the bridge is automatically and rapidly balanced in discrete steps. The sequence of steps is prescribed by simple digital logic and is independent of the magnitude of the bridge output. The bridge output is used for only one purpose, to determine the direction in which the correction should be made.

The known capacitor is a group of n fixed capacitors having values proportional to the binary scale. For example,

$$C_o, 2C_o, 2^2C_o \ldots 2^{n-1}C_o$$

These n capacitors provide 2^n values, and the bridge is balanced by selection of that combination that most nearly balances the bridge. For $n = 9$, each step is 0.2%. For $n = 11$, each step is 0.05%. Capacitor selection is simple and rapid. A binary counter is provided, each level of which controls a switch associated with the capacitor of corresponding level. A "1" in a counter level switches the capacitor in. A "0" switches the capacitor out. Thus, for a counter reading "m," the capacitance connected to the bridge is mC_o.

The counter is driven by a "clock" pulse train. The direction of counting (up or down) is determined by the sign of the output of the bridge. If the digital capacitance is too small, the counter steps up; if too large, the counter steps down. The capacitance changes linearly at the rate of one step per clock pulse until balance is reached, whereupon, without overshoot, the capacitance steps alternately up and down about the point of nearest balance. The general concept is illustrated in the block diagram, Fig. 1.

III. System Properties

The preceding brief description is sufficient to indicate a number of important properties.

Bridge Frequency

The frequency at which the bridge is operated is an important design parameter. In the present system, the bridge frequency is essentially a free choice. It is not constrained, for example, to be suitable for driving an a.c. motor as in many earlier analog servos.[1] There is also the problem in the measurement of pure dielectric liquids of undesirable and unpredictable leakage conduction currents that may flow across the connector and sensor insulators. The information signal is a current proportional to $\omega(\epsilon_L - 1)$. The factor by which the desirable signal exceeds the conduction currents can be made very large by choice of large ω. A good design choice for dielectric liquids such as kerosene, oxygen, and hydrogen is about 100 kc.

Many liquids are not pure dielectrics and in general have an admittivity of the form $\sigma + j\omega\epsilon_L$. It is desirable to operate the bridge so that the phase angle is nearly zero or nearly $\pi/2$. Again the free choice of ω is an important design parameter in achieving the desired condition. This is discussed further in Sec. V.

Output Data

The measurement data from this system are readily available either in digital or analog form. The n voltages at the counter terminals comprise the measurement directly in binary code. This is a rugged data form suitable for transmission or computation without degradation in accuracy. The voltages are approximately 20 v and 0 v for 1 and 0, respectively, and are easily converted to other levels for recording, telemetry, or computer input. It is desirable that the digits not be changing during readout. This can be assured by synchronizing the measuring system clock with the data handling system, or by substituting for the clock an external input signal, or by interrupting the clock for one pulse during readout. The tolerance of the system to a wide range of clock frequencies facilitates such synchronization.

The counter requires a small time interval to respond to an input pulse. This propagation time has a maximum value of about $5 \cdot 10^{-6}$ sec. Thus, the counter is quiescent and conservatively available for readout 10^{-5} sec after a clock input pulse.

An analog output can be obtained by simple weighted-conductance D/A conversion. However, single-scale analog data handling is not usually capable of the fractional percentage accuracy of this system. Multiple scale analog transmission can be provided conveniently by dividing the n digits into several channels of independent D/A conversion. An example is described later of a system ($n = 9$) divided into groups of 4 and 5 digits, respectively. The coarse scale comprises the most significant 4 digits, contains 16 levels, and requires transmission or recording accuracy of ±3.3% or better. The fine scale comprises the 5 least significant digits, contains 32 levels, and requires transmission or recording accuracy of ±1.6% or better. The accuracy requirement can be relaxed further by increasing the number of scales or channels. For example, transmission of three groups of three digits requires an accuracy for each channel of ±7.1%. The extreme case is n channels, which reduces to the binary code in which the accuracy requirement is simple recognition of two states.

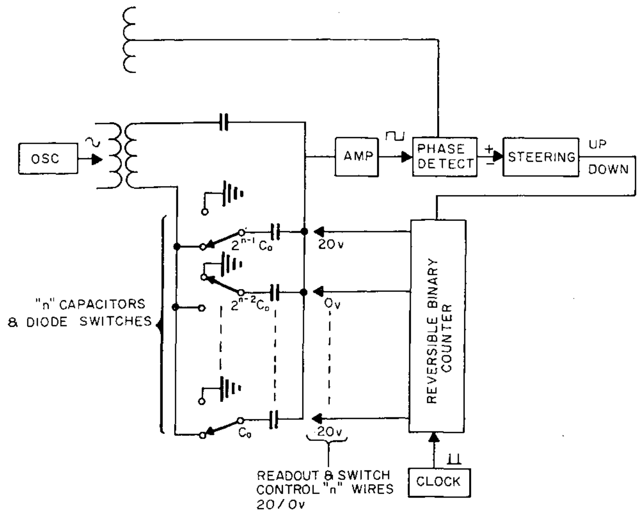

Fig. 1 Basic digital self balancing capacitance system.

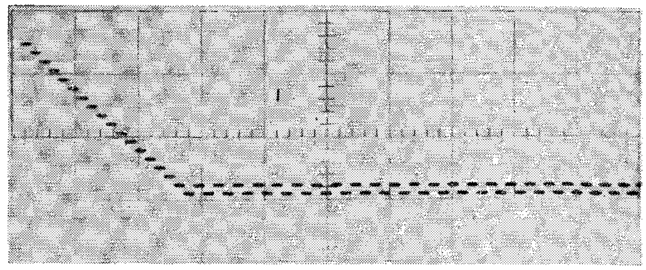

Fig. 2 Transient response to a step decrease in level.

Transient Response

In response to a step change in the unknown capacitance, the digital capacitance bridge changes its output linearly until the new value is reached, whereupon, without overshoot, it alternates in value about the balance point. (See Fig. 2.) Thus, the system tends to follow the input without undershoot or overshoot, and does so at the rate of one bit per clock pulse. This linear response is in contrast with the usual electromechanical servo in which the rate of correction (neglecting servo dynamics) is proportional to error so that the response is basically exponential. In practice, in the electromechanical servo, inertia and saturation effects complicate the response, and some form of damping compensation is usually needed. In the digital capacitance bridge, the correction rate is the clock frequency, and it does not diminish as the error diminishes. Balancing proceeds at the rate of one bit per clock pulse. The only action of the error signal is to provide the direction of correction.

The time required to accomplish a correction is simply $\Delta t = \Delta x/f$. In comparison, a servo in which the response is exponential requires a correction time given by $\Delta t = T \ln \Delta x$. This type of correction tends to be more rapid for large errors but slower for small errors and is less effective in following a rapidly changing input.

The system passes abruptly from transient response to the steady state condition. In the steady state, the bridge capacitance is balanced to within the least significant bit, and since this error is sufficient to actuate the correction loop, the least significant bit is continually switched between states at the clock frequency. If the unknown capacitance is intermediate between integral values of the digital capacitance, the bridge alternates plus and minus one least significant bit. The desired resolution determines the number of digits for which the system is designed. In the case of an electromechanical servo, resolution in the steady state is determined by potentiometer resolution, back lash in the gear train, amplifier drift and gain, etc.

Time Sharing

Between clock pulses there is a time interval $1/f$, which may be a millisecond or more. Throughout this period, the reading is stored in the counter. Part of the time is used to permit the switching transient to decay and part for the sampling pulses. The latter require 10 μsec each, and ten of these require a total of only 0.1 msec. Thus, much of the time is inactive, and during this period the sensors can be switched elsewhere for other measurements. An example is given in Sec. V.

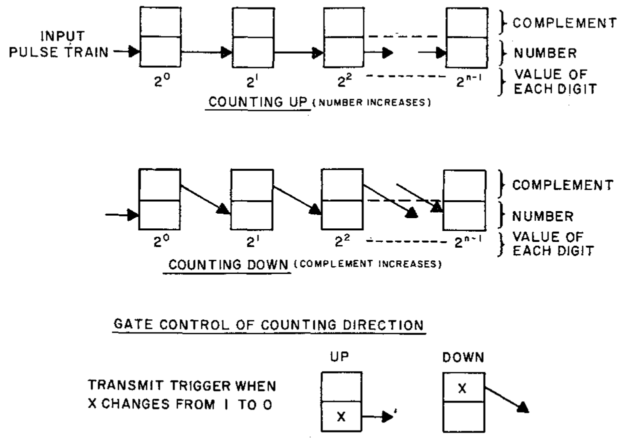

Fig. 3 Reversible counter.

Accuracy

The accuracy and stability of the system are determined by the passive bridge components. To a first order, the accuracy and stability are independent of the characteristics of the other parts of the system. Thus, the system is free from exacting requirements on reference voltage levels, a.c. input signal amplitude and frequency, amplifier gain, and linearity.

IV. Circuits

Reversible Counter

The counter comprises n bistable stages. Upon an input trigger the two terminals of each stage interchange the values 0 and 1. The array consisting of one terminal of each stage represents the number and the array of the other terminals represents the complement.‡ The counter is caused to increase or decrease its number by causing either the number or the complement to increase.

When a terminal of a stage changes from 1 to 0, it transmits a trigger to the following stage. When all such triggers are taken from the number terminal, an input pulse increases the number and the counter counts up; when all such triggers are taken from the complement terminals, an input pulse increases the complement and the counter counts down (Fig. 3). Control of the direction of counting is effected by voltage commands to gates between stages of the counter.

Amplifier and Steering

The phase of the bridge output is either 0 or π relative to the oscillator drive. The amplifier is required simply to raise the signal level so that the phase detector and steering circuit can generate suitable d.c. voltages to control the direction of counting. The amplifier must handle a range of signal magnitudes from full unbalance of the bridge to the smallest unbalance to be resolved. This large dynamic range $\sim 2^n$ is accommodated by providing limiting in the amplifier without phase shift.

Noise is not a consideration for mass measurements because the bridge output signals are relatively large. For level measurements, signals are smaller and a low noise amplifier is necessary for high resolution.

Switches

Both transistor and diode switches have been used in these systems for switching the digital capacitors, for segmenting the sensor, for test circuits, and for time sharing. The diode switch used for switching the digital capacitors consists of two diodes and a simple bias and control circuit (Fig. 4). This circuit is equivalent to a single-pole double throw relay. When the control voltage is less than 1 v, Cr1 is conducting, Cr2 is nonconducting, transmission is maximum, and the switch is "on." When the control voltage is greater than 17v, Cr1, is open, blocking signal transmission, Cr2 is conducting, providing a powerful attenuation ratio, and the switch is "off."

The diodes Cr3 and Cr4 provide the bias for switching Cr1 and Cr2, whereas Cr5 is a zener diode establishing the control level at which switching occurs. The circuit is arranged to provide approximately 1 ma of current through the conducting diode, and approximately 2.1 v of reverse bias across the nonconducting diode.

The dynamic resistance of a conducting diode at room temperature is approximately

$$R_{\text{a.c.}} = 48/I \cdot T/300$$

where I is the forward current in milliamperes. The curve (Fig. 5) showing dynamic resistance as a function of current was derived from published static characteristics. For a given temperature, static curves illustrating the tolerance in forward characteristics reduce to the same dynamic resistance curve, showing that this is a function only of temperature and current. Thus, at 1 ma, the conducting diode has a resistance of approximately 48 ohms. The nonconducting diode is reverse-biased by the forward voltage drop across three diodes (Cr_3, Cr_4, and whichever of Cr1 or Cr2 is conducting). This reverse bias is of the order of 2.1 v. At this bias, the diode appears to be approximately a resistance of 10^8 ohms shunted by a capacitance of 5 pf.

The performance of the switch is shown in Fig. 4. The effect of temperature is negligible over a wide range, and the control voltages are not critical. The attenuation of the "open" switch is greater than 70 db, and that of the closed switch is less than 0.1 db for a 3000-ohm load.

Waveforms

Operation of a digital capacitance system is illustrated in the oscilloscope traces shown in Figs. 2, 6, and 7. These were taken using a 9 bit level system designed for use with either kerosene or oxygen. Provision was made for a two-

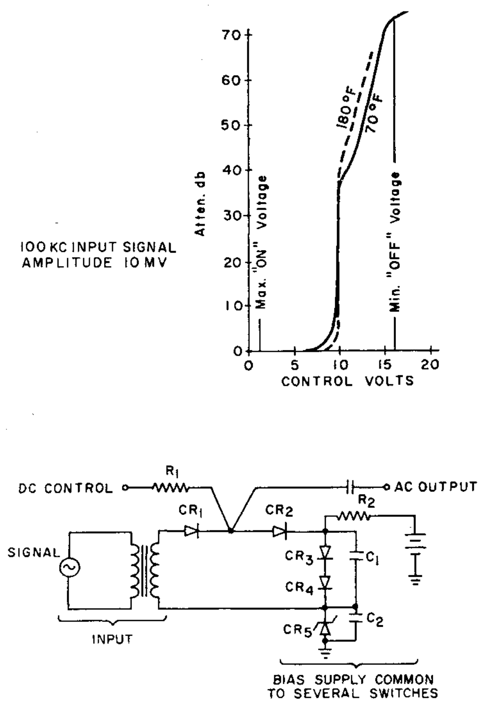

Fig. 4 Diode switch performance.

‡ The sum of the number and the complement is the capacity of the counter.

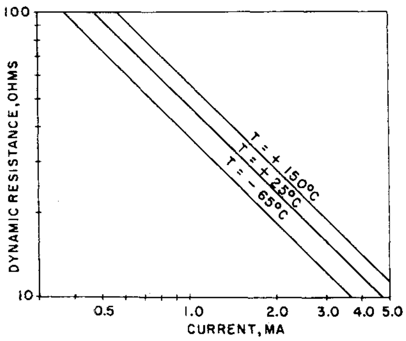

Fig. 5 Dynamic resistance type IN482A diodes vs forward bias and temperature.

scale analog output; the most significant four bits comprising a coarse scale, the least significant five bits comprising a fine scale. Phase detection was performed by a narrow sampling pulse derived from the bridge oscillator. The bridge and clock frequencies were approximately 100 kc and 700 cycles, respectively. Figure 6 shows the analog output signals, both coarse and fine scales, as the system responds to a simulated rising liquid level. Each step of the coarse scale represents 3.2 in., and each step of the fine scale represents 0.10 in.

Figures 2 and 7 both show the response to a step in liquid level. The Fig. 7 trace shows the 100 kc bridge output after some amplification. When the step is first applied, the amplifier is limiting. A few milliseconds later, the a.c. envelope can be seen approaching a null in discrete steps, finally reaching a small steady-state amplitude. The transients caused by switching are visible on this trace. Figure 6 shows the corresponding analog output trace. This output is a staircase to the steady-state level. At the steady-state level, the output is seen to alternate between two adjacent levels. The liquid level is between these two indicated levels but, in this example, is not midway between them. Thus, the unbalance signals in the two adjacent digital positions are unequal as is clearly shown in Fig. 7.

V. Application to Propellant Quantity Measurements

Among the important applications of the digital self-balancing capacitance system are measurements of quantity of rocket propellants. The following several examples have been selected from this field. The first three deal with dielectric fluids and the fourth case with conducting fluids.

Mass

A fundamental molecular property of dielectric liquids provides a stable and accurate means for measuring the mass of liquid that partially fills a tank. Each molecule has a known mass and also makes a known contribution to electric polarization in accordance with the linear relationship $p = aE$. For the bulk liquid that fills the space between the electrodes of a capacitor, $(\epsilon - 1)/(\epsilon + 2) = b\rho$, and for a limited range of density, $\epsilon - 1 = k_\rho \rho$. This relationship holds accurately

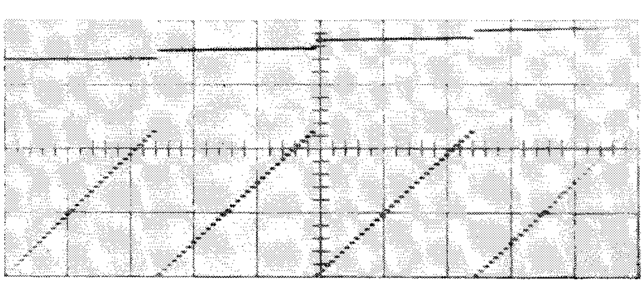

Fig. 6 Coarse and fine scale analog outputs.

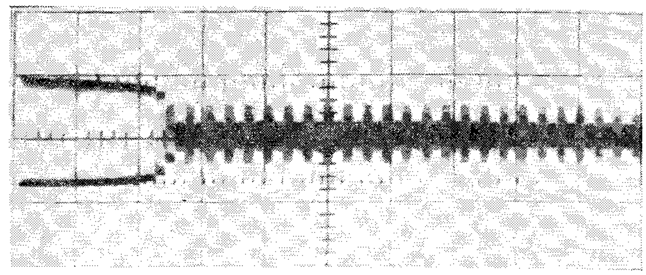

Fig. 7 Transient response to a step decrease in level (amplified bridge output).

for pure dielectric liquids such as oxygen and hydrogen and is independent of temperature. A measuring capacitor or sensor in the tank runs from the bottom to the maximum height of interest. The walls of the sensor are perforated, and it is open at the bottom so that it is filled with liquid to the same height as the rest of the tank.

For an element along the length of such a capacitor

$$\frac{dC}{dh} - \frac{dC_v}{dh} = k_\rho \rho \frac{dC_v}{dh}$$

Integration of this equation along the dimension parallel to the axis of the tank gives the total capacitance

$$C - C_v = k_\rho \int_0^{h_s} \rho_{(h)} \frac{dC_v}{dh} dh$$

The term $\rho_{(h)}$ provides an integration of axial density distribution and the volume of the tank is accounted for by providing $dC/dh = k_a A_{(h)}$. Thus,

$$C - C_v = k_a k_\rho \left\{ \int_0^{h_L} \rho_{L(h)} A_{(h)} dh + \int_{h_L}^{H} \rho_{g(h)} A_{(h)} dh \right\} = k_a k_\rho (M_L + M_g)$$

The quantity $C - C_v$ is a measure of the mass of fluid in the tank, independent of level, average temperature, or axial temperature distribution. It does not account for radial temperature distribution.

It remains to measure $C - C_v$, and the digital capacitance bridge can be applied directly. The subtraction $-C_v$ is accomplished by a capacitor in the bridge C_v'.

The bridge circuit is indicated in Fig. 8, and the bridge equation is

$$k_a k_\rho (M_L + M_g) + C_v = nC_0 + C_v'$$

and the desired fluid mass is given by

$$M_L + M_g = n(C_0/k_a k_\rho)$$

The capacitor C_v' is required to equal C_v rather precisely. For example, let $C_v' = C_v(1 + \delta)$. Then the fractional full scale mass error is

$$[\delta C_v/(\epsilon_L - 1)C_v] = \delta/(\epsilon_L - 1)$$

Suppose the tanks of oxygen and hydrogen containing masses

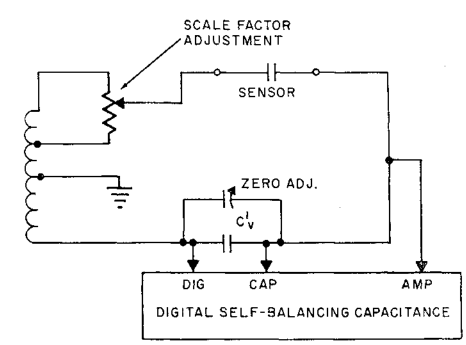

Fig. 8 Application to measurement of fluid mass.

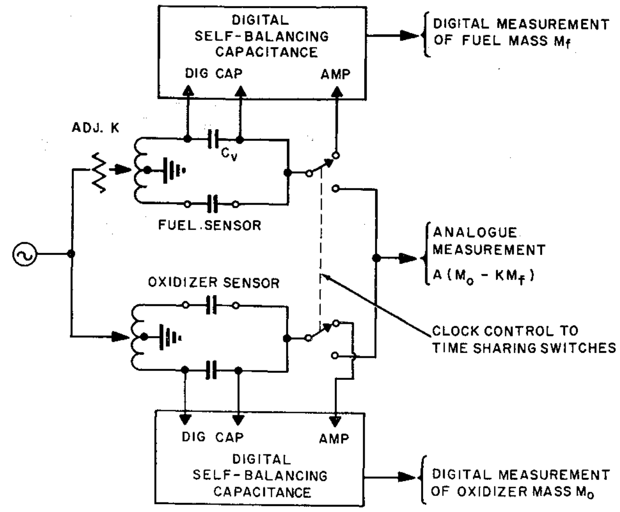

Fig. 9 Example of time shearing of sensors between two measurements.

in the ratio 5–1 are being measured, that a total error of 0.05% is permitted for this quantity, and that the two capacitors have the same fractional change:

$$(\delta_0/0.48)\cdot\tfrac{5}{6} + (\delta_h/0.23) - \tfrac{1}{6} = 5 \times 10^{-4}$$

$$\delta_0 = \delta_h = 2 \times 10^{-4}$$

For capacitors with a temperature coefficient of $10^{-5}/°C$, this corresponds to a permitted temperature band of $\pm 20°C$.

The signal levels are sufficiently high here so that noise is not a problem. Consider, for example, an 11 bit system for measuring hydrogen in which 10 v is applied to each side of the bridge. The output of the bridge for an unbalance of $\frac{1}{2}$ bit is

$$\frac{1}{2}\cdot\frac{e_i}{2}\cdot\frac{n}{1 + 1/(\epsilon_L - 1)}$$

This corresponds to 0.23×10^{-3}v. For oxygen the voltage is even larger.

PU/mass time sharing

In Rocket PU control[2] there is a requirement for the measurement of normalized mass difference $M_o - KM_f$. This is readily accomplished in a bridge circuit which contains the oxidizer and fuel sensors. There is the additional and somewhat conflicting requirement for measurement of the individual masses M_o and M_f, for which connection of the sensors to two independent bridges is desirable. There are several solutions. Independent bridges for M_o and M_f with subsequent subtraction for the PU control signal is feasible, but it is not attractive either for accuracy or reliability. A second solution is to use a single bridge for the analog PU signal connecting to its output a very small load resistance so that current measurements can be made of the individual masses M_o and M_f. The third possibility, that of time sharing the sensors between a PU control bridge and two independent mass bridges, avoids the requirement of the small load resistance and the current measurements and provides optimum independent bridges for the three measurements. There is a comment on this method in Sec. III and a block diagram in Fig. 9.

Level and Volume

The level or volume of liquid in a tank can be measured by the ratio of two capacitances; one a sensor in the region of measurement, and one completely submerged. The ratio of capacitances provides a measurement independent of average density or dielectric constant. It may be used over a wide range of temperature and for a variety of liquids. The system can be extended to conducting liquids by measuring the ratio of admittances.§ An axially uniform sensor measures level; a shaped sensor $dC_v/dh = k_a A_{(h)}$ accounts for tank volume.

Two situations will be discussed. The first (short sensor) is suitable where the density gradient is a small effect, because either the propellant density is uniform, or because the sensor is short and the intercepted density difference is small. The second method (segmented sensor) is suitable for high accuracy measurements over relatively long tanks and where gradients may be more severe.

Short Sensor System

The system is described quantitatively by considering an axially uniform capacitor filled to height h with liquid of dielectric constant ϵ. The capacitance is given by

$$C_m = C_{mv}[\epsilon_L(h/H) + \epsilon_g(H - h/H)]$$

or

$$C_m - C_{mv} = C_{mv}[(\epsilon_L - \epsilon_g)(h/H) + \epsilon_g - 1]$$

Similarly, the capacitance of the submerged reference capacitor is

$$C_r - C_{rv} = C_{rv}(\epsilon_L - 1)$$

These equations combine to give the desired height measurements as follows:

$$\frac{h}{H} = \frac{C_m - C_{mv}}{C_r - C_{rv}}\cdot\frac{C_{rv}}{C_{mv}}\cdot(1 + \alpha_g) - \alpha_g$$

The quantity C_{rv}/C_{mv} is a fixed scale factor and $\alpha_g = (\epsilon_g - 1)/(\epsilon_L - \epsilon_g)$ is a gas correction that is negligible for most room temperature liquids but may be significant for cryogenic liquids. In the latter case adequate compensation can often be provided either in the sensor design or by computation. For 100 psi helium over kerosene, α_g is approximately 0.05%. For gaseous oxygen at 120 k and 2 atm pressure over liquid oxygen at 83 k, α_g is approximately 0.48%. The uncertainty in α is much smaller.

It remains to measure the ratio $(C_m - C_{mv})/(C_r - C_{rv})$ and this is effectively accomplished by the digital capacitance bridge. The bridge circuit is indicated in Fig. 10. C_1 and C_2 are chosen equal to C_{rv} and C_{mv}, respectively. The operation of this bridge is described by the following equations:

$$i_1 = j\omega e(C_r - C_1) \qquad i_2 = j\omega e(C_m - C_2)$$

$$e_1 = -\omega^2 L_{12} e(C_r - C_1) \qquad e_2 = \omega^2 L_{12} e(C_m - C_2)$$

$$e_3 = 0 = \frac{ne_1 + Ne_2}{2N} \qquad \frac{C_m - C_2}{C_r - C_1} = \frac{n}{N}$$

$$\frac{h}{H} = \frac{n}{N}\cdot\frac{C_{rv}}{C_{mv}}(1 + \delta_g) - \delta_g$$

In this bridge measurement of ratio, the attenuation is greater than in the previous bridge measurement of capacitance. Thus, unlike the case for mass measurement, some care is necessary in design of the amplifier input circuit to keep the noise level safely below the error signal.

Segmented Sensor System‖

The sensor consists of a number of identical segments assembled to achieve the desired total length. Discrete level detectors located at the segment junctions identify that segment that intersects the liquid surface. The capacitance of

§ See the subsection on conducting fluids.

‖ This concept is an extension of the idea embodied in Ref. 3.

that segment is measured by a self-balancing digital capacitance.

The bridge design is indicated in Fig. 11. The even numbered segments are connected in parallel to one side of the bridge and the odd numbered segments to the other side. Thus, the sensor arms of the bridge are balanced when an even number of segments are submerged and are at maximum unbalance when an odd number are submerged. A digital capacitor connected across one arm of the bridge maintains balance and measures the value of ΔC for the partially submerged segment (or the complement of ΔC). Since ΔC is proportional to the height of liquid in the segment, the digital capacitance has a value proportional to h for even numbered segments and to $H - h$ for odd numbered segments. The total liquid height is given by

$$H_m = n_d H + \frac{C_D - C_{n,n-1}}{C_{n+1,n} - C_{n,n-1}} H$$

The presence of gas in the upper portion of the sensor and a density gradient in the liquid are accounted for as follows. The balance condition is

$$C_1 + C_3 + \ldots + C_{N-1} = C_2 + C_4 + \ldots + C_N + C_D$$

and the capacitance of a segment is

$$C_n = C_v + K \left\{ \alpha_L \rho_L \frac{h}{H} + \alpha_g \rho_g \left(1 - \frac{h}{H}\right) \right\}$$

where the approximation $\epsilon - 1 = \alpha\rho$ has been used. For constant density gradients in both liquid and gas,

$$\rho_{Li} - \rho_{i(i+1)} = \Delta\rho_L$$

$$\rho_{gi} - \rho_{g(i+1)} = \Delta\rho_g$$

giving the balance condition for odd and even numbered segments, respectively,

$$C_D = k\alpha_L \frac{n-1}{2} \Delta\rho_L + k(\alpha_L \rho_{LL} - \alpha_g \rho_{gn}) \frac{h}{H} + k\alpha_g \frac{N - (n-1)}{2} \Delta\rho_g$$

$$C_D = k\alpha_L \frac{m}{2} \Delta\rho_L + k(\alpha_2 \rho_{Lm} - \alpha g \rho_{gm}) \times \left(1 - \frac{h}{H}\right) + k\alpha_g \frac{N - m}{2} \Delta\rho_g$$

As these equations show, the value of C_D at a segment junction differs slightly from zero due to density gradients in both liquid and gas. The full scale value of C_D depends additionally upon the liquid and gas densities near the liquid surface but not upon the densities elsewhere.

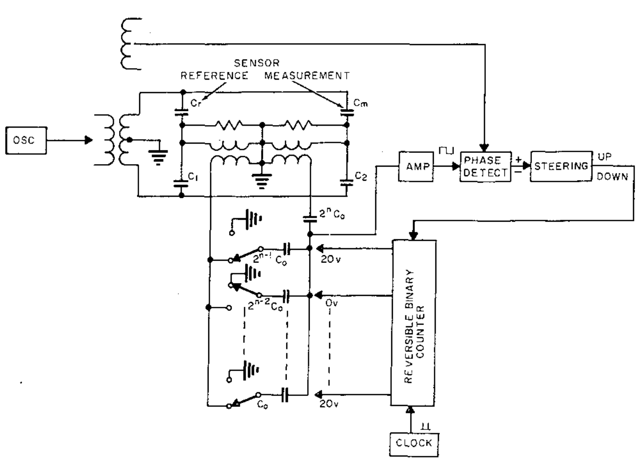

Fig. 10 Application to measurement of liquid level.

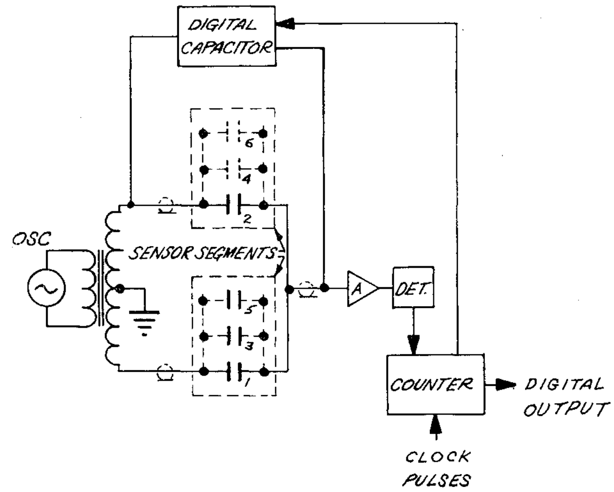

Fig. 11 Segmented sensor system.

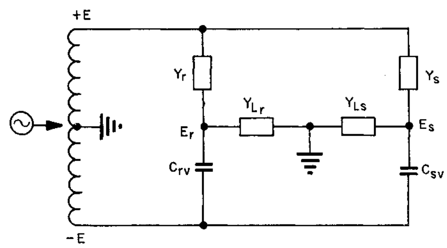

Fig. 12 Admittance bridge.

During draining or filling, the zero and full scale values of C_D are identified for each segment by the discrete level detectors thereby making the system selfcalibrating. The calibrating values contain additional useful information. For example, the average value of the liquid density gradient $(\Delta\rho_L/H)$ over the submerged part of the sensor can be computed as can the liquid density (ρ_{Ln}) averaged over segment n.

Conducting Liquids

Propellants can be divided into two major categories: 1) liquids that are good dielectrics such as O_2 and H_2 having known composition, and RP_1 having variable composition, and 2) liquids that are partly or principally conducting such as N_2O_4 and A_{50}. To classify these, consider the admittance Y of a sensor:

$$dY = \frac{dI}{V} = \frac{(I + \dot{D})}{E} F dh$$

where

$F = W/d$ for parallel plate
$F = (2\pi/\ln r_1/r_2)$ for coaxial, etc.

$$Y = \underbrace{\int_0^h [\sigma + j\omega(\epsilon_L - 1)\epsilon_0] F dh}_{\text{fluid property}} + \underbrace{\int_0^{hm} j\omega\epsilon_0 F dh}_{\text{empty susceptance}}$$

The propellant categories are classified by

$$\text{dielectric} \quad \frac{\sigma}{\omega(\epsilon_L - 1)\epsilon_0} 1 \ll \qquad \text{conducting} \quad \frac{\sigma}{\omega(\epsilon_L - 1)\epsilon_0} \gg 1$$

The measurement of level and volume of dielectric fluids discussed earlier with reference to Fig. 10 is now generalized to the admittance case with reference to Fig. 12. Here Y_s and Y_r are the admittances of the measuring and reference sensors, respectively:

$$\frac{E_s}{E} = \frac{Y_s}{Y_s + Y_{Ls}} = \frac{y_f F h}{y_f F h + Y_{Ls}} = \frac{xe^{i\varphi}}{1 + xe^{i\varphi}}$$

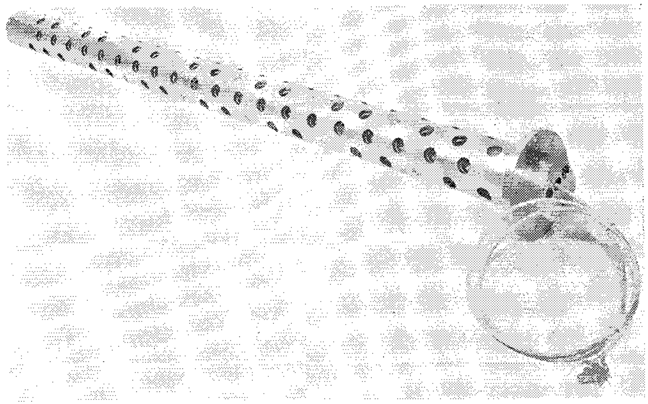

Fig. 13 Sensor for measurement of propellant level (residual oxygen & kerosene in Saturn S1 booster).

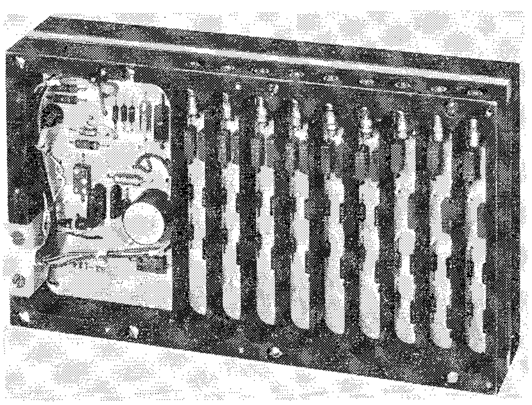

Fig. 14 Bridge component showing digital capacitor.

where

$$x = Fh|y/Y_{Ls}| \qquad \varphi = \angle_y/Y_{Ls}$$

The quantity x is proportional to liquid height and to the admittivity of the propellant. As in the ratio bridge discussed for the dielectric case, the dependence upon propellant admittivity will be cancelled by a similar dependence in the reference sensor. The linearity and stability of the system are dependant upon the nominal value of φ and the maximum permitted value of x. Of the several possibilities of good choice is the following:

$$\varphi \approx \frac{\pi}{2} \qquad \frac{E_s}{E} = \frac{x(i + x)}{1 + 2x\delta\varphi + x^2}$$

$$\mathrm{Im}\,\frac{E_s}{E} = \frac{x}{1 + 2x\delta\varphi + x^2}$$

This equation for the $\mathrm{Im}E_s/E$ brings out the conditions that must be met for linearity. The denominator must be unity within the tolerance on linearity and this leads to simple criteria for $x_{\max}$ and for $\delta\varphi$. The quantity $\delta\varphi$ is in turn a

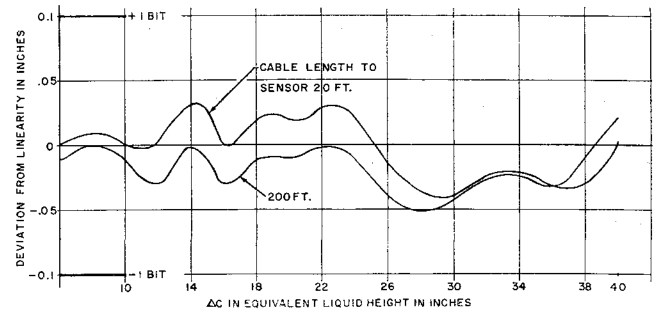

Fig. 15 Linearity of electronic unit for two cable lengths (9 bit level system).

criterion for the tolerable range of propellant properties and for the optimum choice of ω.

VI. Example of a Nine Bit Level System

An example of the techniques described in this paper is a system designed for measurement of residual propellants in the fuel and oxygen tanks of the Saturn S-1 Booster. Level is measured over a length of 40 in., each bit corresponding to 0.10 in. Two scale analog and digital outputs are provided. The equipments used for kerosene and oxygen are identical, except that the oxygen units are adjusted to compensate for the combined effects of shrinkage of the sensor at low temperature and the density of the cold ullage gas.

Two equipments comprise the system: the sensor in the tank (Fig. 13) and an electronic unit external to the tank. The sensor consists of coaxial cylinders riveted together. Insulated spacers are electrically isolated by guard plates. The reference capacitor is colinear with the measuring capacitor and both are provided with guard rings to obtain linearity in the capacitance function of height. Generous perforation of the cylinders assures rapid draining. The electronic unit consists of plug-in printed circuit boards in a water proof case. Figure 14 illustrates the bridge unit showing the digital capacitor. Adjustments are provided in the bridge for zero and full scale. All other boards are interchangeable without adjustment. Both sensor and electronic units have been tested successfully under vibration at an amplitude of 20g up to 2000 cycles.

A record of the calibration of an electronic unit is shown in Fig. 15. It illustrates the linearity of this design for cable lengths of 20 and 200 ft.

References

[1] Allen, D. J. and Bekemeyer, L. G., "Design of the Saturn S-IV stage propellant utilization system," Inst. Radio Engrs. Preprint 1292 (March 1962).

[2] Blanchard, R. L., Sherburne, A. E., and Scott, R. E., "Instrumentation for loading and inflight utilization of liquid propellants in missiles and spacecraft," presented at National Telemetering Conference, Chicago, Ill. (May 1961).

[3] Greenwood, T. L. and Schuler, A. E., "Liquid level measurement system," patent 2, 996, 915 (August 22, 1961).